Pseudo-supersymmetry and the domain-wall/cosmology correspondence

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Abstract
The correspondence between domain-wall and cosmological solutions of gravity coupled to scalar fields is explained. Any domain-wall solutions that admit a Killing spinor are shown to correspond to a cosmology that admits a pseudo-Killing spinor; whereas the Killing spinor obeys a Dirac-type equation with Hermitian 'mass'-matrix, the corresponding pseudo-Killing spinor obeys a Dirac-type equation with a anti-Hermitian 'mass'-matrix. We comment on some implications of (pseudo)supersymmetry.

PACS numbers: 98.80.−k, 11.30.Pb, 11.27.+d

1. Introduction

Domain-wall solutions of supergravity theories, in spacetime dimension $D = d + 1$, have been intensively studied in recent years because of their relevance to gauge theories via holographic renormalization. Initial studies concentrated on the case for which the D-dimensional spacetime is foliated by d-dimensional Minkowski spaces; in other words, flat domain walls. More recently, attention has been focused on curved domain walls, specifically those foliated by d-dimensional anti-de Sitter (adS) spacetimes, although domain-wall solutions foliated by d-dimensional de Sitter (dS) spacetimes have also been considered; we shall refer to these as '(a)dS-sliced' domain walls. In all these cases, the maximal symmetry of the 'slices' implies that only scalar fields are relevant to the solution, so the general low-energy Lagrangian density of interest takes the form

$$\mathcal{L} = \sqrt{-\det g}\left[R - \tfrac{1}{2}|\partial\Phi|^2 - V(\Phi)\right], \tag{1.1}$$

for metric g, with scalar curvature R, and scalar fields Φ taking values in some Riemannian target space and with potential energy function V.

In the supergravity context, models of this type arise as consistent truncations, and a solution for which the supersymmetry variation of all fermion fields vanishes for nonzero supersymmetry spinor parameter ϵ is said to be 'supersymmetric'. The vanishing of the supersymmetry variation of the gravitino field leads to a 'Killing spinor' equation of the form $\mathcal{D}\epsilon = 0$, where $\mathcal{D}$ is an exterior covariant derivative on spinors constructed from the standard spin connection and a 'superpotential', which is (generically) a multi-component function of the scalar fields that determines the potential V through a simple derivative formula. For domain-wall solutions it turns out that the constraints on ϵ implied by the vanishing supersymmetry variations of other fermion fields are algebraic integrability conditions for the differential Killing spinor equation, so they yield nothing new. Thus, supersymmetric domain-wall solutions are those for which $\mathcal{D}\epsilon = 0$ can be solved for some nonzero spinor ϵ, which is called a 'Killing spinor'.

One reason for interest in supersymmetric solutions of a supergravity theory, in particular supersymmetric domain-wall solutions, is that supersymmetry implies stability; in particular, it implies classical stability. However, classical stability cannot depend on the fermionic field content; instead, it depends only on the existence of a Killing spinor. This is a weaker condition than supersymmetry since, for example, the existence of a Killing spinor places no restriction on the spacetime dimension D. This suggests a weaker definition of supersymmetry, which has become known as 'fake' supersymmetry, according to which a solution is considered 'supersymmetric' if it admits a Killing spinor. However, the concept of fake supersymmetry depends on an understanding of what constitutes a Killing spinor outside the supergravity context. There is no general definition, as far as we are aware, but if we restrict our attention to domain-wall solutions then the problem can be reduced, essentially, to a specification of the restrictions to be imposed on the superpotential used to define the covariant derivative operator $\mathcal{D}$. As we shall see, there is an ambiguity in this supergravity-independent definition of a Killing spinor, and one of our purposes here is to explain how this ambiguity may be exploited in the context of cosmology.

In cosmology, the requirement of homogeneity and isotropy implies, just as for domain walls, that the only relevant fields other than the metric tensor are scalar fields, so the Lagrangian density (1.1) also provides a general starting point for the study of Friedmann–Lemaître–Robertson–Walker (FLRW) cosmologies. In fact, there is a correspondence between domain-wall solutions and FLRW cosmologies for these models. For every domain-wall solution of the model with scalar potential V there is a cosmological solution of the same model but with scalar potential $-V$, and vice versa [1]. Here we present the details of the analytic continuations that connect the domain-wall and cosmological spacetimes that are paired by this 'domain-wall/cosmology correspondence'. Special cases have been noted on many previous occasions but the generality of the correspondence seems not to have been hitherto appreciated. It raises the question of how special features of domain walls, such as supersymmetry, are to be interpreted in the context of cosmology. This point was addressed briefly in [1]: cosmologies that correspond to supersymmetric domain walls are 'pseudo-supersymmetric' in the sense that they admit a 'pseudo-Killing' spinor. The possibility of pseudo-Killing spinors arises precisely from the above-noted ambiguity in the extension of the notion of a Killing spinor to fake supersymmetry. Here we explain this point in more detail and discuss possible implications of pseudo-supersymmetry for cosmology.

2. Domain-wall/cosmology correspondence

The D-dimensional spacetime metric for a d-dimensional domain wall of maximal symmetry can be put into the form

$$ds_D^2 = dz^2 + e^{2\beta\varphi} \left[-\frac{d\tau^2}{1 + k\tau^2} + \tau^2 \, d\Omega_+^2 \right], \tag{2.1}$$

where we have introduced the D-dependent constant

$$\beta = 1/\sqrt{2(D-1)(D-2)}, \tag{2.2}$$

and $d\Omega_+^2$ is an $SO(1, d-1)$-invariant metric on the unit radius d-dimensional hyperboloid; we may choose coordinates such that

$$d\Omega_+^2 = d\psi^2 + \sinh^2 \psi \, d\Omega_{d-2}^2. \tag{2.3}$$

A domain-wall spacetime is therefore determined by a scale function $\varphi(z)$ and a constant k, which we may restrict to the values $0, \pm 1$, without loss of generality. The local geometry of the (d-dimensional) hypersufaces of constant z is de Sitter for $k = 1$, Minkowski for $k = 0$, and anti-de Sitter for $k = -1$. For a given 'fiducial' choice of z, these spacetimes can be viewed as the wall's 'worldvolume'; the coordinate z is thus a measure of distance from this fiducial worldvolume. In order to preserve the local isometries of the wall's worldvolume, the scalar fields Φ must be restricted to be functions of z only.

Leaving aside domain walls for the moment, we turn to cosmology. The D-dimensional spacetime metric for an FLRW cosmology has the form

$$ds_D^2 = -dt^2 + e^{2\beta\phi} \left[\frac{dr^2}{1 - kr^2} + r^2 \, d\Omega_-^2 \right] \tag{2.4}$$

where $d\Omega_-^2$ is an $SO(d)$-invariant metric on the unit radius d-sphere; we may choose coordinates such that

$$d\Omega_-^2 = d\theta^2 + \sin^2 \theta \, d\Omega_{d-2}^2. \tag{2.5}$$

FLRW cosmologies are therefore determined by the scale function $\phi(t)$ and the constant k, which we may again restrict to the values $0, \pm 1$, without loss of generality. The (d-dimensional) constant t hypersurfaces are spheres for $k = 1$, Euclidean spaces for $k = 0$ and hyperboloids for $k = -1$, corresponding to closed, flat and open FLRW universes, respectively. In order to preserve homogeneity and isotropy, the scalar fields Φ must be restricted to be functions of t only.

The above domain-wall and cosmological spacetimes are related by analytic continuation. To see this, we start from the domain-wall spacetime of (2.1), define the new variables

$$(t, r, \theta) = -i(z, \tau, \psi), \tag{2.6}$$

and then analytically continue to real values of (t, r, θ). This yields the FLRW metric (2.4) if we define

$$\phi(t) = \varphi(it). \tag{2.7}$$

This makes it appear that ϕ is a complex function of t but it is actually a real function that solves the field equations of the model with the opposite sign of both V and k. A simple example is adS_D sliced by adS_d, d-dimensional Minkowski, or dS_d spacetimes, which become the $k = 1, 0, -1$ representations of dS_D as FLRW universes. For solutions that involve the scalar fields Φ, one must similarly reinterpret the functions $\Phi(z)$ of the domain wall spacetime as real functions of t, which we then rename (in a slight abuse of notation) as $\Phi(t)$. The reason that this analytic continuation always works, in the sense that the real functions determining a domain wall solution become real functions determining a cosmological solution, can be seen as follows [1].

Let us consider the domain-wall and cosmological solutions together by introducing a sign η such that $\eta = 1$ for domain walls and $\eta = -1$ for cosmologies. Then, in either case, the metric can be put in the form

$$ds_D^2 = \eta(e^{\alpha\varphi} f)^2 \, dz^2 + e^{2\beta\varphi} \left[-\frac{\eta \, d\tau^2}{1 + \eta k \tau^2} + \tau^2 \, d\Omega_\eta^2 \right], \tag{2.8}$$

where

$$\alpha = (D-1)\beta = \sqrt{\frac{D-1}{2(D-2)}}, \tag{2.9}$$

and, in order to maintain z-reparametrization invariance, we have made the replacement $dz \to e^{\alpha\varphi(z)} f(z) \, dz$ for (lapse) function f, which must be monotonic but is otherwise arbitrary; the gauge choice $f = e^{-\alpha\varphi}$ yields the forms of the domain-wall or cosmological metrics given above. The scalar fields Φ are functions only of z, which is a space coordinate for $\eta = 1$ and a time coordinate for $\eta = -1$. The Euler–Lagrange equations of (1.1) then reduce to equations for the variables (φ, Φ) that are equivalent to the Euler–Lagrange equations of the effective Lagrangian

$$L = \frac{1}{2} f^{-1}(\dot{\varphi}^2 - |\dot{\Phi}|^2) - f \, e^{2\alpha\varphi} \left(\eta V(\Phi) - \frac{\eta k}{2\beta^2} \, e^{-2\beta\varphi} \right), \tag{2.10}$$

where the overdot indicates differentiation with respect to z. It follows immediately from the form of this effective Lagrangian that for every solution of the $\eta = 1$ equations of motion for potential V there is a corresponding solution of the $\eta = -1$ equations of motion with potential $-V$, with the opposite sign of k if $k \neq 0$, and vice versa. The domain-wall and cosmological solutions paired in this way are precisely those related by the analytic continuation procedure described above.

3. Fake supersymmetry

For a single scalar σ, the effective Lagrangian (2.10) reduces to

$$L = \frac{1}{2} f^{-1}(\dot{\varphi}^2 - \dot{\sigma}^2) - f \, e^{2\alpha\varphi} \left(\eta V(\sigma) - \frac{\eta k}{2\beta^2} \, e^{-2\beta\varphi} \right). \tag{3.1}$$

It was observed in [3] that the choice of target spaces coordinates can always be adapted to any given solution in such a way that this solution involves only a single scalar field, so the restriction to a single scalar is much less severe than one might suppose. For this reason, many properties of single-scalar solutions can be extended to multi-scalar solutions. However, there are subtleties that arise in the application of this idea that we do not wish to enter into here, so we restrict ourselves to the one-scalar case.

An example of a supergravity model with a single scalar field is the pure minimal $D = 5$ gauged supergravity, and this provides a convenient, as well as physically relevant and historically significant, starting point for a study of fake supersymmetry of domain walls. The superpotential of this model is a real $SU(2)$ triplet $\mathbf{W}$ and a straightforward generalization of the Killing spinor equation for this model suggests that we choose the exterior covariant derivative $\mathcal{D}$, mentioned in the introduction, to be [4]

$$\mathcal{D} = dx^\mu [D_\mu + \alpha\beta \mathbf{W} \cdot \boldsymbol{\tau} \Gamma_\mu], \tag{3.2}$$

where D_μ is the standard covariant derivative operator acting on Dirac spinors, $\boldsymbol{\tau}$ is the triplet of Pauli matrices acting on $SU(2)$ spinors, and Γ_μ are the spacetime Dirac matrices. By this definition, $\mathcal{D}$ acts on $SU(2)$ doublets of Lorentz spinors. These would satisfy a

symplectic reality condition in the context of minimal $D = 5$ supergravity but, in the spirit of fake supersymmetry, we relax this condition here. The factor of $\alpha\beta$ arises from a choice of normalization of $\mathbf{W}$; this is fixed by the relation between $\mathbf{W}$ and V, which in our conventions is

$$V = 2[|\mathbf{W}'|^2 - \alpha^2|\mathbf{W}|^2], \tag{3.3}$$

where the prime indicates a derivative with respect to σ.

For a domain-wall metric of the form (2.1) (i.e. for gauge choice $f = \mathrm{e}^{-\alpha\varphi}$), the Killing spinor equation implies

$$\partial_z \epsilon = \alpha\beta\mathbf{W} \cdot \boldsymbol{\tau}\Gamma_{\underline{z}}\epsilon, \tag{3.4}$$

where $\Gamma_{\underline{z}}$ is a *constant* matrix that squares to the identity, and

$$\hat{D}\epsilon = \mathrm{e}^{\beta\varphi}\hat{\Gamma}[(\beta/2)\dot{\varphi}\Gamma_{\underline{z}} + \alpha\beta\mathbf{W} \cdot \boldsymbol{\tau}]\epsilon, \tag{3.5}$$

where $\hat{D}$ is the standard worldvolume exterior covariant derivative on spinors and $\hat{\Gamma}$ is the *worldvolume* Dirac matrix valued 1-form.

The integrability conditions for these equations were discussed in detail in [5] and a simplified analysis was presented in [1]. We will not repeat the full analysis here, but we note that (3.5) has the integrability condition

$$\dot{\varphi}^2 = 4\alpha^2|\mathbf{W}|^2 + (k/\beta^2)\,\mathrm{e}^{-2\beta\varphi}, \tag{3.6}$$

which upon use of the field equations leads to

$$\dot{\sigma} = \pm 2|\mathbf{W}'|. \tag{3.7}$$

The joint integrability condition of (3.4) and (3.5) is

$$(\dot{\sigma} + 2\mathbf{W}' \cdot \boldsymbol{\tau}\Gamma_{\underline{z}})\epsilon = 0, \tag{3.8}$$

which can be interpreted in the supergravity context as the condition arising from the vanishing of the supersymmetry variation of the super-partner of σ. This and (3.7) then lead to the projection equation

$$(1 \pm \Gamma)\epsilon = 0, \qquad \Gamma = \frac{\mathbf{W}' \cdot \boldsymbol{\tau}}{|\mathbf{W}'|}\Gamma_{\underline{z}} \tag{3.9}$$

so the domain wall is half supersymmetric. In general, this equation has its own integrability condition, since Γ is a function of z, and this implies that

$$(\mathbf{W}'' + \alpha\beta\mathbf{W}) \times \mathbf{W}' = 0. \tag{3.10}$$

From this we deduce that $\mathbf{W}$ must take the form

$$\mathbf{W} = \mathbf{n}\,\mathrm{Re}\,Z(\sigma) + \mathbf{m}\,\mathrm{Im}\,Z(\sigma), \tag{3.11}$$

where $\mathbf{n}$, $\mathbf{m}$ are two orthonormal 3-vectors and Z is a complex function, with $Z = W$ for real scalar function W when $k = 0$. In addition, a complete analysis of consistency requires

$$|\mathbf{W} \times \mathbf{W}'|^2 = -k(D - 2)^2\,\mathrm{e}^{-2\beta\varphi}|\mathbf{W}'|^2. \tag{3.12}$$

It was shown in [1], by direct construction of $Z(\sigma)$, that any $k = 0$ or $k = -1$ domain-wall solution admits a Killing spinor provided that the function $\sigma(z)$ is strictly monotonic; all such solutions are therefore (fake) supersymmetric. The $k = 0$ case is especially simple, and was discussed earlier in [5, 6]. A Hamiltonian perspective on the general construction may be found in [2]. A solution for which $\dot{\sigma}(z)$ has isolated zeros can be considered 'piecewise supersymmetric' but the construction breaks down completely if the zeros of $\dot{\sigma}(z)$

accumulate. As shown in [2], unstable adS vacua are accumulation points and hence domain-wall spacetimes that are asymptotic to an unstable adS vacuum are not (fake) supersymmetric, as expected since they are also unstable. If we agree, for the sake of simplicity, to leave aside these exceptions, then we can summarize the result by saying that *all flat or adS-sliced walls are (fake) supersymmetric*.

This result was obtained for a particular choice of operator $\mathcal{D}$, so we should consider to what extent it depends on this choice. In the supergravity context, the superpotential is generally in some non-trivial representation of the R-symmetry group that acts on the gravitino field, this being $SU(2)$ for minimal $D = 5$ supergravity. We took this $D = 5$ example, with real $SU(2)$-triplet superpotential, as our starting point and generalized it to arbitrary dimension D, relaxing the symplectic reality condition on the spinor in the process. Recall that one of the implications of integrability is that the real triplet superpotential is actually determined by a complex function Z. The restrictions on Z imposed by (3.10) and (3.12) could have been found directly by taking $D = 4$ minimal supergravity as the starting point because the superpotential in this case is naturally a complex function; this route is less convenient, however, because of the complications of chirality. Note that an attempt to further simplify by assuming a real singlet superpotential (as suggested by $D = 3$ minimal supergravity) would restrict fake supersymmetry to flat domain walls. This is an unnecessary restriction, so it is important to consider whether some more general superpotential might similarly show that the restrictions obtained by the assumption of a real-triplet superpotential are similarly unnecessary. For example, one could consider [7] the USp(4) 5-plet superpotentials suggested by extended $D = 5$ supergravity. It would be surprising if this were to allow new possibilities[3] because the real triplet superpotential is already unnecessarily general; it is determined by a complex function. Moreover, the triplet superpotential is already sufficient to establish the fake supersymmetry of almost all flat or adS-sliced domain walls. There are good physical reasons for the exceptions, as noted above. Nor should we expect to discover that some more general superpotential that will allow the dS-sliced walls to be considered fake-supersymmetric because (for $D > 2$) there is no physically acceptable supersymmetric extension of the de Sitter group. These considerations fall short of being a proof but they convince us that domain walls that are not (fake) supersymmetric in the particular sense described above will not become (fake) supersymmetric for some other choice of the exterior differential operator $\mathcal{D}$.

4. Pseudo-supersymmetry

As already observed, in certain spacetime dimensions it may be possible to impose a symplectic reality condition on ϵ, and this is required for $D = 5$ minimal supergravity since the minimal $D = 5$ spinor is an '$SU(2)$-Majorana' spinor. A symplectic reality condition on ϵ effectively enforces the reality of $\mathbf{W}$ since complex conjugation of the Killing spinor equation $\mathcal{D}\epsilon = 0$ then yields the same equation but with $\mathbf{W}$ replaced by its complex conjugate. Once the symplectic reality condition on ϵ is relaxed, there is no immediate reason why $\mathbf{W}$ should be real, although the reality of V implies that it must be either real or pure imaginary. Of course, if $\mathbf{W}$ is pure imaginary, then we can redefine it to be real at the cost of changing the covariant derivative $\mathcal{D}$ from the expression given in (3.2) to

$$\mathcal{D} = \mathrm{d}x^\mu (D_\mu + i\alpha\beta \mathbf{W} \cdot \boldsymbol{\tau} \Gamma_\mu). \tag{4.1}$$

At the same time, we must change the relation (3.3) to

$$V = -2[|\mathbf{W}'|^2 - \alpha^2 |\mathbf{W}|^2]. \tag{4.2}$$

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[3] Note however that such more general superpotential may be useful in establishing the existence of more than one fake supersymmetry.

This can be viewed as the same relation as (3.3) but for a model with scalar potential of the opposite sign from the original model.

At this point, we see how the ambiguity in the notion of a Killing spinor outside the supergravity context might be exploited in cosmology, because the cosmological 'dual' of a domain-wall solution of a given model with scalar potential V is a solution of the same model but with V replaced by $-V$. For the FLRW metric (2.4) the use of (4.1) yields the equations

$$\partial_t \epsilon = \alpha \beta \mathbf{W} \cdot \boldsymbol{\tau} \mathrm{i} \Gamma_{\underline{t}} \epsilon, \tag{4.3}$$

where $\Gamma_{\underline{t}}$ is a *constant* matrix that squares to *minus* the identity, and

$$\hat{D}_a \epsilon = \mathrm{e}^{\beta \varphi} \mathrm{i} \hat{\Gamma}_a [-(\beta/2) \dot{\varphi} \mathrm{i} \Gamma_{\underline{t}} + \alpha \beta \mathbf{W} \cdot \boldsymbol{\tau}] \epsilon, \tag{4.4}$$

where Γ_a are the Dirac matrices, in a coordinate basis, for a fiducial spacelike hypersurface of fixed t. This has the integrability condition

$$\dot{\varphi}^2 = 4\alpha^2 |\mathbf{W}|^2 - (k/\beta^2) \, \mathrm{e}^{-2\beta\varphi}. \tag{4.5}$$

This is the same as (3.6) if we take $k \to -k$, which is a consequence of the overall factor of i on the right-hand side of (4.4). The joint integrability conditions of (4.3) and (4.4) is

$$(\dot{\sigma} + 2\mathbf{W}' \cdot \boldsymbol{\tau} \mathrm{i} \Gamma_{\underline{t}}) \epsilon = 0. \tag{4.6}$$

Because $\mathrm{i}\Gamma_{\underline{t}}$ squares to the identity, this is equivalent to (3.8) and hence leads to an equivalent constraint on ϵ and, for $k \neq 0$, equivalent constraints on $\mathbf{W}$. To summarize, given a domain-wall solution with $k = 0, -1$ there is a construction of a real triplet superpotential $\mathbf{W}$ such that the wall admits a Killing spinor. The corresponding $k = 0, 1$ cosmological solution of the model with $V \to -V$ then admits a spinor satisfying a similar equation but with $\mathbf{W} \to \mathrm{i}\mathbf{W}$. We shall call such a spinor a 'pseudo-Killing' spinor. The difference between Killing and pseudo-Killing spinors can be characterized as follows: by taking the 'gamma-trace' of the (pseudo-)Killing spinor equation we deduce that the (pseudo-)Killing spinor satisfies a Dirac-type equation of the form

$$\slashed{\mathcal{D}} \epsilon = M \epsilon, \tag{4.7}$$

where M is a 'mass' matrix (albeit a non-constant one). For a genuine Killing spinor this mass matrix is Hermitian whereas for a pseudo-Killing spinor it is anti-Hermitian.

We have now seen how to construct a pseudo-Killing spinor for a cosmological solution of a model of the type defined by (1.1) starting from a Killing spinor associated with a supersymmetric domain-wall solution of the same model but with the opposite sign potential. Given that almost all flat or adS-sliced domain-wall solutions are supersymmetric, we may now conclude that almost all flat or closed FLRW cosmologies are 'pseudo-supersymmetric'. Perhaps the simplest, although very special, example of a supersymmetric domain wall is a stable adS vacuum. The dS spacetime that is its cosmological 'dual' is then pseudo-supersymmetric. In the following section, we shall explore some implications of this fact.

5. Applications

Anti de Sitter space can be viewed as a special case of a flat domain-wall spacetime. It can also be viewed as either an adS-sliced or a dS-sliced domain wall, but the standard Minkowski slicing will be sufficient for present purposes. Given a potential $V(\sigma)$, maximally symmetric vacua correspond to constant values of σ for which $V(\sigma)$ is extremized. Let us suppose that V has an extremum at $\sigma = 0$ with $\eta V_0 < 0$, so that the vacuum is adS_D for $\eta = 1$ and dS_D for $\eta = -1$. In this case V has the Taylor expansion

$$V = -\frac{\eta}{2\beta^2 \ell^2} + \frac{1}{2} m^2 \sigma^2 + \mathcal{O}(\sigma^3) \tag{5.1}$$

where ℓ is the (a)dS radius and m is the mass of the scalar field fluctuation. Let V be given by

$$\eta V = 2[(W')^2 - \alpha^2 W^2], \tag{5.2}$$

for real singlet superpotential W. This is just (3.3) with $\mathbf{W}$ given by (3.11) with $Z = W$. As mentioned earlier, a real singlet superpotential suffices for consideration of flat domain walls, and can be found for any V, at least in principle, by solving the differential equation (5.2) for W. We therefore have

$$\eta V' = 4W'(W'' - \alpha^2 W), \tag{5.3}$$

from which we see that there are two types of (a)dS vacua. Those for which $W' = 0$ and those for which $W' \neq 0$ (in which case $W'' = \alpha^2 W$). The adS vacua with $W' = 0$ are the supersymmetric vacua; this terminology is consistent with our earlier terminology for domain walls because when σ is constant and ϵ is nonzero, the supersymmetry preserving condition (3.8) reduces to $W' = 0$. Similarly, the dS vacua with $W' = 0$ are the pseudo-supersymmetric vacua. By differentiating (5.3) and evaluating at the stationary point of W, one can derive a bound on m^2. For $\eta = 1$, this is the Breitenlohner–Freedman (BF) bound [8, 9]

$$m^2 \geqslant -\frac{(D-1)^2}{4\ell^2}, \tag{5.4}$$

which states that m cannot be 'too tachyonic', and the method of proof is that of [10, 11]. The BF bound is not absolute because it is a trivial matter to construct a model with an adS vacuum that violates the bound; these vacua are not associated with stationary points of W and are not supersymmetric. Nevertheless, adS vacua that satisfy the bound are physically distinct from those that do not; the former are stable, at least classically, whereas the latter are unstable.

The same procedure leads, for $\eta = -1$, to a cosmological analogue of the BF bound. This is the *upper* bound [2]

$$m^2 \leqslant \frac{(D-1)^2}{4\ell^2}. \tag{5.5}$$

Again, the bound is not absolute but serves to separate dS vacua with distinct physical properties. If this inequality is satisfied then the scalar field σ approaches its equilibrium value at the dS vacua monotonically, like an overdamped pendulum. If the potential rises 'too steeply' from its (positive) minimum, such that the bound is violated, then σ will overshoot its equilibrium value and then oscillate about it *ad infinitum* as it approaches this value, like an underdamped pendulum. This implies that a dS vacuum violating the bound is an accumulation point for zeros of $\dot{\sigma}$. The same is true of adS vacua that violate the BF bound and this is why domain walls that are asymptotic to unstable adS vacua fail to be (fake) supersymmetric. In the cosmological case, however, it is less clear that a violation of the bound implies an instability. One might expect the oscillations implied by a violation of the bound to cause a particle production that could reduce the potential energy of the dS vacuum. Instabilities of dS space due to particle production have been proposed [12] but also opposed [13]. We will not attempt to review the current situation here; it appears from a recent analysis [14] that the matter is still not completely resolved.

Acknowledgments

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